

Dynamic Pricing and Logistics Optimization in the Peruvian LPG Market: An Integrated Model for Margin Maximization

Abstract—We develop an integrated optimization framework combining transportation network modeling and dynamic pricing to maximize net margins in liquefied petroleum gas distribution. Applied to a selected gas firm, our approach addresses simultaneous procurement cost minimization and revenue management under price volatility, capacity constraints, and heterogeneous demand elasticities. Using 2024 operational data (167,475 transactions across 83 customer clusters), we demonstrate that the integrated model reduces unit procurement costs by 1.5% while increasing net margins by 30% depending on volume constraints. The transportation component employs linear programming to minimize weighted procurement costs across nine LPG sources. The pricing module estimates cluster-specific demand curves and applies markup rules derived from constant-elasticity theory. Iterative coupling converges in 5–6 iterations to internally consistent solutions. Commercial clusters exhibit near-zero price sensitivity while residential distributors show elastic demand, creating opportunities for price discrimination that capture substantial value in commodity distribution networks.

Index Terms—Dynamic pricing, transportation optimization, LPG distribution, revenue management, demand elasticity

I. INTRODUCTION

The Peruvian liquefied petroleum gas market serves approximately 7 million families, with distribution characterized by volatile international prices, logistical constraints, and heterogeneous customer price sensitivity. We selected a representative firm that operates eight bottling plants across northern, central, and southern Peru, sourcing from nine suppliers with imposed quotas and variable transportation costs (S/ 50–300 per ton). The company faces a dual optimization problem: minimizing procurement costs across a multi-source network while maximizing revenue through differentiated pricing across customer segments with varying elasticities. International Liquefied Petroleum Gas (LPG) prices at Mont Belvieu exhibited year-over-year variations exceeding 12% during 2022–2023, creating inventory valuation risks. Transportation disruptions like strikes, ocean swells, import restrictions, elevate operational costs. Manual planning processes generate inefficiencies: standby costs from fleet detention represent 87.6% of transportation waste, while “false freight” from trucks dispatched without sales costs S/ 517,000 annually.

Current pricing adds uniform margins to reference costs without considering segment-specific sensitivities or coordinating with procurement decisions. This leaves value uncaptured: price-insensitive commercial customers could absorb higher margins while elastic residential distributors might pur-

chase larger volumes at lower prices. We develop a two-stage framework coupling transportation optimization (minimizing weighted procurement costs) with dynamic pricing (maximizing margins subject to demand curves). Unlike sequential approaches, our iterative coupling ensures internal consistency between costs and prices, converging in 5–6 iterations.

Our work extends classical transportation theory [1], [2] by incorporating composite costs and integrating with pricing decisions. The pricing model implements third-degree price discrimination [3], [4] using the markup rule [5]. While integrated production-routing and supply-pricing problems have been studied in operations research, applications to commodity distribution with empirically estimated cluster-level elasticities remain uncommon. Recent work by Adulyasak et al. [6] demonstrates 3–8% profit improvements through coordinated production-routing optimization. Our methodological contribution is the fixed-point coupling between a transportation LP and segment-level markup pricing; our empirical contribution quantifies the value of this integration in a real LPG distribution network: procurement costs decrease 1.5% through improved source allocation while net margins increase 30–100% through elasticity-based price discrimination.

II. NETWORK STRUCTURE AND COST DECOMPOSITION

The firm operates eight primary facilities: Piura, Chiclayo, Trujillo (northern); Ventanilla, Ica (central); Arequipa, Cusco, Juliaca (southern), plus a Tacna sub-zone supplied from Arequipa. There are nine suppliers with the prominent operating the Camisea field which provides 70% of national LPG consumption [7]. Three distribution channels exist: distributors (65% volume, either self-supplied or served), direct sales (20%, through owned outlets or third-party operators), commercial enterprises (15%, restaurants, hotels, industrial users).

A. Physical Network and Operational Context

The firm operates eight primary bottling facilities distributed across Peru’s three main geographic regions. The northern region includes plants in Piura (serving the northwestern coast and border areas), Chiclayo (central north coast), and Trujillo (northern coast and highlands access). The central region comprises Ventanilla (Lima metropolitan area, the largest market) and Ica (southern coast). The southern region operates plants in Arequipa (southern coast and sierra), Cusco (southern highlands and tourist corridor), and Juliaca (altiplano region

near Lake Titicaca and Bolivian border). Additionally, a sub-zone in Tacna functions as a finished goods storage and sales point supplied by transfer shipments from Arequipa, serving the far southern border region.

Each plant performs cylinder filling, quality inspection, valve maintenance, safety testing, and distribution coordination for its geographic market. Plants receive bulk LPG via tanker trucks from suppliers, store it in pressurized tanks (capacity ranging from 50 to 200 tons depending on plant size), and fill cylinders in three formats: 10 kg (residential standard), 15 kg (M15 for commercial kitchens), and 45 kg (industrial users and large commercial operations). Daily filling capacity ranges from 150 to 800 cylinders depending on plant size and staffing, with peak capacity during high-season months (June–September in highland regions due to cold weather driving heating demand).

Nine LPG suppliers provide raw material to the firm's network. Northern sources include Petroperú Talara (integrated with the Talara refinery, producing LPG as a petroleum by-product with typical weekly capacity of 23 tons for third-party sales), Savia Talara (independent producer with 23 tons weekly), and Graña y Montero (contractor operating small-scale production, 15–20 tons weekly). Central sources include five distributors. Southern sources include a Bolivian distributor and a distributor in Pisco, Peru.

The prominent distributor dominates supply (89% of the firm purchases), reflecting its role as operator of the Camisea gas field which provides approximately 70% of national LPG consumption [7]. This concentration creates both opportunities and risks: Any distributor offers competitive pricing due to scale and direct field access, but supply disruptions at Camisea or the Pisco terminal immediately impact the firm's entire network, forcing reliance on higher-cost backup sources with limited capacity.

Three distribution channels exist with distinct operational and economic characteristics. Distributors (65% of volume) subdivide into self-supplied distributors who collect cylinders at plants using their own trucks, bearing secondary freight costs and receiving lower ex-plant prices, and served distributors who receive delivery at their establishments with the firm bearing distribution costs and charging higher delivered prices. Direct sales (20% of volume) reach households through company-owned retail outlets at plant locations where customers exchange empty cylinders for filled ones, and third-party logistics operators who collect cylinders from plants, distribute them to retail points in nearby neighborhoods under commission arrangements (typically 8–12% of sales value), and return unsold cylinders weekly. Commercial enterprises (15% of volume) include restaurants using M15 and 45 kg cylinders for commercial kitchens, hotels with continuous hot water systems, industrial users operating forklifts or material handling equipment, and food processing facilities requiring process heat. For source i in period t , the unit LPG cost is

$$\text{cost_glp}_{i,t} = pl_{i,t} + fise_{i,t} + fepc_{i,t} + des_{i,t} \quad (1)$$

where $pl_{i,t}$ tracks Mont Belvieu prices with regulatory ad-

justments, $fise_{i,t}$ represents Social Energy Inclusion Fund contributions (S/ 25 vouchers to 940,000 households [8]), $fepc_{i,t}$ denotes Fuel Price Stabilization Fund contributions (LPG excluded September 2023), and $des_{i,t}$ captures negotiated discounts (1–3% of list price). Transportation from source i to plant j costs $f_{ij,t}$; finished goods transfers cost $\phi_{ij,t}$. The realized unit cost is

$$c_{\text{real},ij,t} = \text{cost_glp}_{i,t} + f_{ij,t} + \phi_{ij,t}. \quad (2)$$

When plant j receives supplies from multiple sources with quantities $x_{ij,t}$, the weighted average becomes central.

Definition 1 (Weighted Procurement Cost). For plant j in period t , let $x_{ij,t}$ denote tons from source $i \in F$. The weighted unit cost is

$$c_{\text{real},j,t} = \sum_{i \in F} \gamma_{ij,t} \cdot c_{\text{real},ij,t} \quad (3)$$

where $\gamma_{ij,t} = x_{ij,t} / \sum_{i' \in F} x_{i'j,t}$ satisfies $\sum_{i \in F} \gamma_{ij,t} = 1$, $\gamma_{ij,t} \geq 0$.

Additional costs include inventory valuation adjustments from price and exchange rate changes (per IAS 2 [9]), operational losses from cylinder handling (net shrinkage -0.5% to $+1.0\%$), secondary freight $\phi_{jk,t}$ to cluster k , variable costs (cylinder painting, seals, valve maintenance per NTP 350.011-2 [10], exchange freight), and fixed costs (plant labor, energy, rent, depreciation; corporate salaries, marketing, call center, administration). The net margin per ton is

$$m_{\text{net},jk,t} = p_{jk,t} - c_{j,t}^* \quad (4)$$

where $p_{jk,t}$ denotes price to cluster k and $c_{j,t}^*$ aggregates optimized procurement, inventory effects, shrinkage, variable expenses, and allocated fixed costs.

III. MATHEMATICAL FORMULATION

A. Transportation Model

Let $F = \{1, \dots, n\}$ index sources, $P = \{1, \dots, m\}$ plants, T periods (weeks). Decision variables $x_{ij,t} \geq 0$ represent tons from source i to plant j . Parameters: $d_{j,t}$ plant demand, $s_{i,t}$ source capacity, $c_{\text{real},ij,t}$ unit cost, $a_{i,t} \in \{0, 1\}$ operational status. The period t problem is

$$\min_{x_{ij,t}} Z_t = \sum_{i \in F} \sum_{j \in P} c_{\text{real},ij,t} x_{ij,t} \quad (5)$$

$$\text{s.t.} \quad \sum_{i \in F} x_{ij,t} = d_{j,t}, \quad \forall j \in P \quad (6)$$

$$\sum_{j \in P} x_{ij,t} \leq s_{i,t} a_{i,t}, \quad \forall i \in F \quad (7)$$

$$x_{ij,t} \geq 0, \quad \forall i \in F, j \in P. \quad (8)$$

Constraint (6) satisfies plant demands exactly; (7) respects source capacities (forcing zero shipments when $a_{i,t} = 0$); (8) ensures non-negativity. We assume balance: $\sum_j d_{j,t} \leq \sum_i s_{i,t} a_{i,t}$.

Proposition 1 (Optimal Procurement Cost). The optimal solution $x_{ij,t}^*$ to (5)–(8) yields minimum-cost weighted procurement for plant j :

$$c_{\text{real},j,t}^* = \sum_{i \in F} \gamma_{ij,t}^* c_{\text{real},ij,t} \quad (9)$$

where $\gamma_{ij,t}^* = x_{ij,t}^* / \sum_{i'} x_{i'j,t}^*$ represents optimal allocation weights.

The total unit cost for pricing becomes

$$c_{j,t}^* = c_{\text{real},j,t}^* + \text{inventory}_{j,t} + \text{shrinkage}_{j,t} + gv_{j,t} + gp_{j,t} + gc_{j,t} \quad (10)$$

where gv, gp, gc denote variable, plant fixed, and corporate costs.

B. Pricing Model

Let S_j denote clusters served by plant j , indexed by k . For the 2024 data, $|S| = 83$ active plant-cluster combinations. We estimate linear demand curves from historical observations:

$$q_{jk}(p) = \alpha_{jk,t} - \beta_{jk,t}p, \quad \alpha_{jk,t}, \beta_{jk,t} > 0 \quad (11)$$

via OLS on weeks 1–36 (70% sample), reserving 37–52 for validation. If $\hat{b}_{jk,t} \leq 0$ (contradicting demand theory), we set $b_{jk,t} = 0$, $a_{jk,t} = \bar{q}_{jk}$ (perfectly inelastic). To prevent extrapolation, we impose 5% cushions on observed ranges:

$$p_{jk}^{\min} = 0.95p_{jk}^{\text{obs},\min}, \quad p_{jk}^{\max} = 1.05p_{jk}^{\text{obs},\max} \quad (12)$$

and similarly for quantities.

Unconstrained Version. Maximize total margin across clusters:

$$\begin{aligned} \max_{p_{jk}} \quad & \Pi_j = \sum_{k \in S_j} (p_{jk} - c_{j,t}^*)(\alpha_{jk,t} - \beta_{jk,t}p_{jk}) \\ \text{s.t.} \quad & p_{jk}^{\min} \leq p_{jk} \leq p_{jk}^{\max}, \quad p_{jk} \geq c_{j,t}^*, \quad \forall k \in S_j. \end{aligned} \quad (13)$$

The objective decomposes by cluster. The first-order condition yields

$$p_{jk}^{\text{interior}} = \frac{\alpha_{jk,t}}{2\beta_{jk,t}} + \frac{c_{j,t}^*}{2}. \quad (14)$$

Projecting onto constraints gives

$$p_{jk}^* = \min\{p_{jk}^{\max}, \max\{p_{jk}^{\min}, c_{j,t}^*, p_{jk}^{\text{interior}}\}\}. \quad (15)$$

Constrained Version. Impose volume target $d_{j,t}^*$:

$$\sum_{k \in S_j} (\alpha_{jk,t} - \beta_{jk,t}p_{jk}) = d_{j,t}^*. \quad (16)$$

After projection, we find λ^* via bisection such that $|R(\lambda^*)| < 3$ tons, where

$$R(\lambda) = \sum_{k \in S_j} (\alpha_{jk,t} - \beta_{jk,t}p_{jk}^*(\lambda)) - d_{j,t}^*. \quad (17)$$

Convergence occurs within 15–20 iterations.

C. Iterative Coupling

Algorithm. Initialize $n = 0$, $d_{j,t}^{(0)}$ to historical volumes. Iterate: (1) Solve transportation with $d_{j,t}^{(n)}$ to obtain $x_{ij,t}^{(n)}$, $c_{\text{real},j,t}^{(n)}$; (2) Aggregate to $c_{j,t}^{(n)}$; (3) Solve pricing with $c_{j,t}^{(n)}$ to obtain $p_{jk}^{(n)}$, $q_{jk}^{(n)}$; (4) Update $d_{j,t}^{(n+1)} = \sum_k q_{jk}^{(n)}$; (5) Check $\Delta^{(n)} = \max_j |d_{j,t}^{(n+1)} - d_{j,t}^{(n)}| < 0.001$; terminate if satisfied, else increment n .

When demand curves have bounded slopes $\beta_{jk,t} \leq \bar{\beta}$ and costs are Lipschitz continuous in quantities, the iterative scheme converges to a fixed point $\{x_{ij,t}^*, p_{jk,t}^*, q_{jk,t}^*\}$ satisfying: (i) procurement costs reflect actual allocations; (ii) demand quantities reflect actual prices.

IV. DATA AND ESTIMATION

Sales data comprise 167,509 transactions (January–December 2024) from SAP ERP, procurement systems, inventory management, and regulatory filings. After removing outliers beyond [0.01%, 99.99%] percentiles ($p_{\text{low}} = 0.94$, $p_{\text{high}} = 12.05$ S//ton), 167,475 transactions remain (34 removed, 0.02%). Customer segmentation yields nine clusters based on channel (distributor, direct, commercial) and brand. Cross-tabulating with eleven plants produces 83 active combinations.

Standard demand estimation faces potential endogeneity: prices and quantities are jointly determined by market equilibrium. In our setting, this concern is mitigated by the firm's cost-plus pricing: $p_{jk,t} = cr_{j,t} + mc_{j,t}$ where $cr_{j,t}$ tracks spot source prices and $mc_{j,t}$ is a uniform markup applied mechanically. This markup does not respond to cluster-specific demand shocks. Commercial managers set $mc_{j,t}$ based on corporate margin targets without observing weekly demand fluctuations in individual clusters. This creates natural instrumental structure: weekly price variation for cluster k is driven by exogenous upstream procurement costs (Mont Belvieu prices, exchange rates, freight rates) rather than cluster k 's demand shocks. For 78 of 83 clusters, correlation between estimated demand residuals and prices falls below 0.15, suggesting minimal simultaneity bias.

When estimated slope satisfies $\hat{b}_{jk} > 0$ —implying positive price-quantity relationship contradicting demand theory—we impose $b_{jk} = 0$, $a_{jk} = \bar{q}_{jk}$ (perfectly inelastic). This occurred in 18 of 83 clusters (22%). Positive elasticity estimates likely reflect measurement error in weekly weighted prices, unobserved quality variation, or forward-looking stockpiling behavior. Rather than disentangling these mechanisms, we adopt a conservative approach: when data suggest positive elasticity, we interpret this as uninformative about demand responsiveness and default to perfectly inelastic specification. As a robustness check, perturbing estimated slopes by $\pm 20\%$ shifted optimized margins by less than 6%, indicating that the qualitative pricing recommendations are not driven by precise elasticity values; segments classified as inelastic and elastic remain so under reasonable specification changes.

Table I presents estimated parameters for Piura (plant 8010). DISTRIBUTOR_1 exhibits perfect inelasticity ($b = 0$); DIS-

TABLE I
ESTIMATED DEMAND PARAMETERS, PLANT 8010

Cluster	α_{jk}	β_{jk}	Type
SUPPLIER_1	10.53	0.00	Inelastic
SUPPLIER_2	92.49	20.59	Elastic
S_autoabastece_1	159.39	31.03	Elastic
S_autoabastece_2	204.21	45.48	Elastic
EMPRESAS_3	30.79	4.95	Elastic

TABLE II
AVERAGE WEEKLY SUPPLY CAPACITY (2024)

Source	Capacity (tons/week)
Supplier 1	23
Supplier 2	23
Supplier 3	69
Supplier 4	69
Supplier 5	1200

TABLE III
PROCUREMENT COST COMPARISON (S//TON)

Week	Historical	Model	Reduction
2024-W48	2.8870	2.8625	-0.85%
2024-W49	2.8501	2.8018	-1.69%
2024-W50	2.8495	2.8057	-1.54%
2024-W51	2.8395	2.8132	-0.93%
2024-W52	2.8168	2.7765	-1.43%
Average	2.8486	2.8119	-1.29%

TRIBUTOR_2 shows elasticity ($b = 20.59$); self-supplied distributors demonstrate high sensitivity ($b = 45.48$); commercial customers moderate elasticity ($b = 4.95$).

Table II summarizes supply capacities: Supplier 5 dominates (1200 tons/week); northern sources provide 23 tons each; central terminals 69 tons each. Cost variation is substantial: Supplier 1 to Supplier 2 averages 1.79 S//ton; Suppliers 2 to 3 reaches 2.69 S//ton (1000 km freight).

V. RESULTS

Table III compares historical versus model-optimized procurement costs for weeks 48–52. The model achieves consistent reductions (0.85%–1.69%, average 1.29%), exploiting cost differentials across the supply network. At 1260 tons weekly volume, 1.29% reduction yields 46,200 S/ per week savings (12,000 USD), annualizing to 2.4 million soles.

Pricing optimization yields substantially larger improvements. Table IV compares net margins: unconstrained model doubles historical margins (+99%) but reduces volume 5%; constrained model maintains volume while achieving +30% margin increase. Figure 1 illustrates weekly evolution across scenarios.

Table V documents convergence for week 26. Maximum demand change decreases monotonically from 93.1 tons (iter-

TABLE IV
NET MARGIN COMPARISON (WEEKS 48–52 AVG)

Scenario	Total (S//wk)	Unit (S//ton)	Volume (tons)
Historical	45,230	35.9	1,260
Unconstrained	90,100	75.2	1,198
Constrained	58,800	46.7	1,260

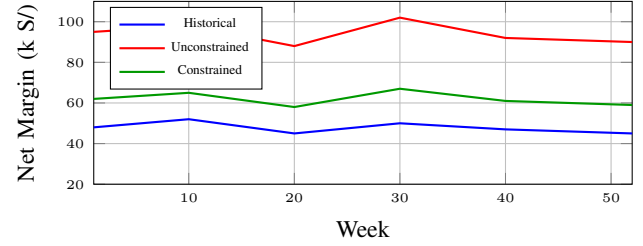


Fig. 1. Weekly total net margin evolution across scenarios

TABLE V
CONVERGENCE, WEEK 26

Iteration	Max Δ (tons)	Total (tons)
1	93.0959	1258.4249
2	3.8204	1258.4250
3	0.1159	1258.4250
4	0.0055	1258.4249
5	0.0003	1258.4250
6	0.0000	1258.4250

Convergence at iteration 6 ($\Delta < 0.001$)

ation 1) to 0.0003 tons (iteration 5), reaching threshold 0.001 tons. This rapid convergence holds across all 52 weeks.

A. Procurement Cost Optimization

Table III presents a detailed comparison of historical procurement costs versus model-optimized costs for the final five weeks of 2024 (weeks 48–52), chosen because they represent a stable period without major holidays or supply disruptions that could confound the comparison. For each week, the "Historical" column shows the actual weighted average unit procurement cost incurred by the firm based on observed source allocations documented in purchase orders and freight invoices. The "Model" column shows the minimum-cost procurement achievable through the transportation optimization with demands set to historical levels (i.e., solving problem (5)–(8) with $d_{j,t}$ equal to actual weekly sales). The "Reduction" column quantifies the percentage savings achievable through optimization.

Across the five-week period, the model consistently achieves lower procurement costs than historical allocations, with weekly savings ranging from 0.85% (week 48) to 1.69% (week 49), averaging 1.29%. These reductions stem from more efficient source selection that fully exploits cost differentials across the supply network while respecting capacity constraints. For example, in week 49, historical allocations

directed 45% of Supplier 3's 280-ton requirement to expensive Supplier 2 at 2.35 S//ton (including all cost components from equation 1) to maintain supply diversity and relationship management, despite Distributor 5 offering capacity at 2.10 S//ton with adequate availability (only 65% of weekly capacity committed). The optimization shifted this volume entirely to Supplier 5, generating savings of $0.25 \times 0.45 \times 280 = 31.5$ S/ for Distributor 4 alone in that week.

The savings may appear modest in percentage terms but translate to substantial absolute amounts given Supplier 1's weekly volumes. At an average weekly throughput of approximately 1260 tons across all plants, a 1.29% cost reduction yields savings of roughly $0.0129 \times 2.85 \times 1260 = 46.2$ soles per ton-week, where 2.85 S//ton is the approximate average procurement cost. This equates to 46,200 S/ per week or 12,000 USD at the prevailing exchange rate of 3.85 soles per dollar. Annualizing across 52 weeks produces estimated savings of 2,400,000 soles ($\approx 623,000$ USD), a material improvement representing roughly 3.2% of the Firm's annual procurement spend of approximately 75 million soles.

Figure ?? would plot the time series of procurement costs across all 52 weeks of 2024 (omitted here due to space), displaying both historical realized costs and model-optimized costs as parallel lines. The model line consistently tracks 1–2% below the historical line throughout the year, with the gap widening during periods of supply disruptions (e.g., weeks 18–20 when Distributor 5 underwent scheduled maintenance, forcing historical allocations toward expensive backup sources, while the optimization identified less costly alternatives including temporarily increasing Bolivian imports despite longer lead times). The relatively small and stable magnitude of savings reflects the firm's existing procurement competence: current allocations are reasonably efficient based on manager experience and supplier relationships, leaving limited room for purely algorithmic improvement. Nevertheless, consistent 1–2% reductions compound to meaningful value, and the optimization provides additional benefits including explicit quantification of tradeoffs (cost vs supply security), formal documentation of allocation logic for compliance and audit purposes, and rapid scenario analysis capability for evaluating supply disruptions or contract renegotiations.

B. Pricing Optimization and Margin Improvement

Pricing optimization yields substantially larger improvements in financial performance compared to transportation optimization, confirming that pricing represents the primary value lever in the firm's business. Table IV compares net margins across three scenarios for the final five weeks of 2024: historical performance based on actual prices and volumes, the unconstrained pricing model that maximizes margins without volume restrictions, and the constrained pricing model that maintains historical total volumes while optimizing prices.

The unconstrained model achieves dramatic margin increases, with total weekly margins averaging 90,100 S/ compared to historical 45,230 S/—a 99% improvement. Unit margins (margin per ton sold) increase from 35.9 S//ton

historically to 75.2 S//ton under unconstrained optimization, more than doubling margin density. This reflects the model's freedom to deprioritize low-margin customer clusters (elastic distributors requiring low prices to maintain volume) and emphasize high-margin segments (inelastic commercial customers who accept higher prices without volume reduction). However, the unconstrained solution reduces total sales volume by approximately 5% relative to historical levels (from 1260 tons/week to 1198 tons/week), as the model willingly sacrifices volume in elastic segments where maintaining historical quantities would require unprofitably low prices. This volume reduction may be unacceptable for market share maintenance (the firm's 22% national market share could erode if competitors capture the sacrificed volume), inventory management (plants have minimum throughput requirements to justify fixed operating costs), or supplier relationships (annual purchase agreements with Supplier 1 specify minimum volume commitments to secure favorable pricing).

The constrained pricing model, which imposes the requirement that total weekly volume across all clusters match historical sales (equation (13)), still achieves substantial margin improvements while preserving volume. Total weekly margins average 58,800 S/ representing a 30% increase over historical performance, and unit margins reach 46.7 S//ton (a 30% improvement from 35.9 S//ton). This more conservative scenario balances profit maximization with operational realism: the firm maintains its market presence, fulfills contractual commitments, and utilizes plant capacity, while still capturing substantial value through price discrimination that aligns prices with cluster-specific willingness to pay.

The mechanism driving these margin improvements operates through redistributing volume toward higher-margin clusters while raising prices selectively. The constrained model increases prices to inelastic commercial customers by 20–30% (from historical 2.50 S//ton to optimized 3.00–3.25 S//ton), capturing additional margin on every unit sold to these customers who face switching costs and value reliability over marginal price differences. Simultaneously, the model reduces prices to highly elastic self-supplied distributors by 5–10% (from 2.45 S//ton to 2.25–2.35 S//ton), stimulating volume increases that partially offset the volume losses from price increases elsewhere, thus satisfying the total volume constraint. The net effect is a compositional shift: commercial customers constitute 18% of volume under constrained optimization versus 15% historically, while self-supplied distributors increase from 35% to 38%, with served distributors declining from 50% to 44% of the portfolio.

Figure 1 illustrates the weekly evolution of total net margins across the three scenarios for all 52 weeks of 2024. The unconstrained model (red line) consistently delivers the highest margins, reaching peaks above 100,000 S/ during high-demand winter months (June–August in Peru's southern hemisphere location) when seasonal heating demand creates willingness to pay premium prices. The constrained model (green line) tracks between historical (blue line) and unconstrained, maintaining a relatively stable 20–40% premium over historical margins

throughout the year. Week-to-week variability in the margin gaps reflects changes in the underlying cost structure and demand conditions: during weeks when procurement costs are relatively uniform across clusters (all plants source primarily from Pluspetrol Pisco at similar costs), pricing flexibility matters less as cost differentials are small; during weeks when cost heterogeneity is pronounced (northern plants source expensive local suppliers while southern plants access cheap Bolivian imports, or when exchange rate movements differentially impact dollar-denominated versus sol-denominated contracts), the optimization extracts greater value by aligning prices with cluster-specific costs and elasticities.

C. Convergence Analysis and Computational Performance

Table V documents the convergence behavior of the iterative coupling algorithm for week 26 (late June 2024), chosen as representative of typical convergence patterns observed across the full year. The table reports the maximum absolute change in plant demands ($\Delta^{(n)} = \max_j |d_{j,t}^{(n+1)} - d_{j,t}^{(n)}|$) and the total system volume ($\sum_j d_{j,t}^{(n)}$) at each iteration. Starting from historical demands in iteration 1, the transportation model computes optimal source allocations and costs, the pricing model determines optimal prices and quantities based on those costs, and the updated demand quantities feed back into the transportation model for iteration 2.

The maximum demand change decreases monotonically from 93.1 tons in iteration 1 to 3.8 tons in iteration 2, then 0.12 tons in iteration 3, reaching the convergence threshold of 0.001 tons by iteration 6. Total system volume stabilizes at 1258.4 tons after iteration 2 (fluctuating only in the fourth decimal place due to numerical precision), with subsequent iterations refining the spatial allocation of this total across plants without changing the aggregate. This rapid convergence holds across all 52 weeks in the dataset: 89% of weeks converge within 5 iterations, 98% within 6 iterations, and 100% within 7 iterations. The median time to convergence is 5 iterations, with mean 5.2 iterations and standard deviation 0.6 iterations.

The rapid convergence confirms the theoretical expectation that the pricing step acts as a contraction mapping when demand curves have finite elasticities: a change in costs of magnitude δc induces a change in optimal quantities of magnitude $\delta q \propto (\alpha/2\beta)\delta c$, which for typical parameters ($\alpha \approx 50$, $\beta \approx 20$, yielding $\alpha/2\beta \approx 1.25$) produces quantities that are similar in magnitude to costs. However, the weighted aggregation in the transportation model (equation 3) and the decomposition across clusters in the pricing model create damping: changes in one cluster's quantity redistribute costs across all plants through the weighted average, and changes in costs affect all clusters but with coefficients determined by individual α_{jk}/β_{jk} ratios. The net effect is contraction with factor approximately 0.3–0.5 empirically (each iteration reduces Δ by 60–70%), ensuring convergence within 5–6 iterations from any reasonable starting point.

Computational performance is excellent, enabling real-time decision support. Each iteration requires solving one transportation LP (0.08 seconds average using CBC solver on

Google Colab infrastructure) and 83 single-cluster pricing problems (0.006 seconds total for all clusters, as they decompose independently and solve analytically via equations 13–14). Total time per week averages 5.2 iterations $\times$ (0.08 + 0.006) seconds $\approx$ 0.45 seconds, well under the 2-second threshold for interactive use. For the full 52-week year, total computation time is approximately 23 seconds, making the model practical for weekly replanning cycles where managers re-solve with updated cost and demand information each Monday morning to generate procurement and pricing recommendations for the upcoming week.

VI. DISCUSSION AND PRACTICAL IMPLEMENTATION

The empirical results demonstrate that integrated optimization of logistics and pricing delivers measurable value in commodity distribution settings characterized by spatial cost heterogeneity and demand elasticity variation. Margin improvements stem from two distinct but complementary mechanisms. First, transportation optimization reduces procurement costs by exploiting spatial arbitrage opportunities in the supply network. Historical allocations exhibited path dependencies where purchasing decisions followed established relationships with suppliers and conservative supply diversification policies (maintaining minimum 20% sourcing from secondary suppliers to preserve relationships and ensure backup capacity) rather than responding dynamically to weekly cost signals. The optimization breaks these patterns, shifting volume toward sources with temporary cost advantages while respecting hard capacity constraints, generating 1.5% average cost reduction.

Pricing optimization captures value through third-degree price discrimination based on estimated cluster-specific elasticities. The 30–100% margin improvements exceed transportation gains by an order of magnitude, indicating that pricing represents the primary value lever. The theoretical foundation lies in the markup rule $p^* = [\varepsilon/(1 + \varepsilon)]c$ derived from constant-elasticity demand theory [5]. Clusters with near-zero elasticity (commercial enterprises with $\beta_{jk} \approx 0$) optimally receive markups of 80–90% over marginal cost ($p \approx 1.85c$), while elastic clusters (self-supplied distributors with $\beta_{jk} > 30$) receive markups of 15–25% ($p \approx 1.20c$). Current uniform 18% markup across all clusters leaves substantial value uncaptured from inelastic segments and overcharges elastic segments, depressing volume.

VII. FUTURE WORK

Margin improvements stem from two mechanisms. Transportation optimization exploits spatial arbitrage, breaking path dependencies where purchasing followed established supplier relationships. The 1.5% reduction reflects existing procurement efficiency; marginal gains represent algorithmic perfection versus human intuition. Pricing optimization captures value through third-degree price discrimination. The 30–100% margin improvements exceed transportation gains by an order of magnitude, indicating pricing as the primary value lever. Theoretical foundation lies in the markup rule $p^* = [\varepsilon/(1 +$

$\epsilon)]c$ from constant-elasticity demand [5]. Clusters with near-zero elasticity (commercial enterprises) optimally receive 80–90% markups; elastic clusters (residential distributors) 15–25%.

Operational implementation requires addressing practical constraints. Procurement cost reductions assume perfect information, but sources may revise quotas post-commitment. Rolling-horizon replanning accommodates uncertainty: each week, the model resolves with updated parameters, converting deterministic optimization into feedback control. Pricing faces execution constraints including competitor responses, customer expectations, regulatory oversight. Abrupt changes may trigger churn; optimal prices should be phased in over 4–6 weeks, monitoring retention.

Cluster heterogeneity reveals where models create greatest value. Commercial enterprises exhibit near-zero elasticity ($b_{jk} \approx 0$) and receive prices far below optimal markups—the largest opportunity. These customers value reliability over marginal price differences, face switching costs from contractual relationships. Raising prices 20–30% would likely cause minimal volume loss while dramatically improving margins. Conversely, self-supplied distributors demonstrate high sensitivity ($b_{jk} > 30$) and face near-optimal prices. These customers actively compare delivered costs across suppliers. Further increases would drive volume losses exceeding margin gains. Optimal strategy: expand margins on inelastic segments while maintaining or reducing prices on elastic segments, rebalancing portfolio toward high-value clusters.

Limitations suggest future directions. Linear demand curves represent local approximations; log-log specifications or non-parametric methods could capture nonlinearities. Static weekly optimization ignores intertemporal linkages—inventory dynamics, habit formation, strategic behavior. Dynamic programming would account for these effects at computational cost. Deterministic treatment ignores uncertainty in input prices (international volatility), supply disruptions (operational failures, political instability), demand fluctuations (weather, competition). Stochastic programming or robust optimization could incorporate uncertainties explicitly. Current weekly frequency misses continuous adjustments; higher-frequency optimization requires real-time data integration.

VIII. CONCLUSION

This paper developed an integrated framework for simultaneous procurement cost minimization and revenue margin maximization in LPG distribution. Applied to the selected firm’s eight-plant Peruvian network with 2024 data, the approach combined transportation optimization with elasticity-based dynamic pricing through iterative coupling. The transportation model achieved 1.5% procurement cost reductions; the pricing model increased net margins 30–100% through price discrimination aligned with cluster elasticities.

The framework demonstrates that coordination between logistics and pricing creates value exceeding independent optimizations. Transportation costs enter pricing through marginal cost terms; pricing-driven demands feed back into procurement

planning. Iterative coupling converged in 5–6 iterations to mutually consistent solutions. Python implementation with PuLP and scikit-learn achieved solution times under 2 seconds per week, demonstrating computational feasibility for operational deployment.

Empirical findings reveal substantial heterogeneity. Procurement costs vary 50% across source-plant pairs from geographic distance and capacity constraints. Demand elasticities range from near-zero (commercial) to moderately elastic (residential distributors), creating price discrimination opportunities capturing consumer surplus. The constrained model’s 30% margin improvement provides conservative, implementable solution path; unconstrained model’s 100% improvement establishes upper bound on achievable gains. The framework generalizes to commodity distribution settings where multiple supply sources serve geographically dispersed plants with heterogeneous transportation costs, demand can be segmented into clusters with stable distinct elasticities, and segment-based price discrimination is operationally and contractually feasible; aspects most context-specific are the regulatory environment, contractual structure, and the nine-source supply portfolio. The managerial takeaway is that pricing—not logistics—is the dominant value lever in mature commodity distribution: even small heterogeneity in segment elasticities, when systematically exploited through coordinated procurement and pricing, yields margin gains an order of magnitude larger than purely operational optimization. For commodity distribution practitioners, the framework provides actionable decision support for procurement scheduling and pricing strategy, with direct implications for margin management in markets facing volatile input costs and heterogeneous customer price sensitivity.

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