

A Binational COIL Matrix-Cryptography Challenge to Teach Invertibility and SDG 4 in Engineering Education

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Abstract— *This qualitative case study examines a binational Collaborative Online International Learning (COIL) experience that uses matrix cryptography to teach matrix inversion in engineering education while connecting disciplinary work to SDG 4 (Quality Education). Forty undergraduates from two Latin American universities (Mexico and Chile) collaborated in mixed teams ($n = 9$) to decrypt a plaintext encoded with a 10×10 key matrix using Python, MATLAB, and Excel, and then produced an evidence–action memo linking the decrypted message to SDG 4 targets and proposing feasible on-campus actions with indicators. Data included team reports ($n = 9$), recorded presentations, and individual reflections ($n = 40$). An inductive codebook thematic analysis supported by Atlas.ti yielded three interrelated dimensions: (i) technical (71 coded segments), with cross-tool validation achieved by 100% of teams; (ii) pedagogical (59 coded segments), with explicit SDG 4 linkage in all team reports; and (iii) emotional–collaborative (63 coded segments), highlighting intercultural motivation and joint problem solving across sources. To provide post hoc evidence of conceptual understanding, we applied a Rubric for Conceptual Understanding of Invertibility (RCCI) to team artifacts: 78% of teams provided complete justification of invertibility conditions, and 67% demonstrated systematic error diagnosis and correction. Although no objective pre/post tests were administered, the findings suggest that cryptography-based COIL tasks can strengthen procedural autonomy, conceptual justification, and socially grounded relevance in linear algebra.*

Keywords— *linear algebra; COIL; cryptography; educational innovation; higher education.*

I. INTRODUCTION

Traditional instruction in linear algebra faces a crisis of relevance. Although AI, data science, and cryptography rely critically on matrix operations and vector spaces, pedagogical approaches remain anchored in abstract, decontextualized exercises. This disconnect produces professionals who master the theory but cannot apply it to real-world problems—such as optimizing machine-learning models, processing images, or designing secure algorithms—limiting their ability to innovate in areas where linear algebra is foundational. Prior work reports persistent difficulties in acquiring linear algebra concepts, including overreliance on procedures and insufficient justification of key properties [1]. This pattern aligns with Sfard's dual-process framework [20], where students treat matrices as computational objects rather than mathematical structures with properties and constraints. This shortcoming is amplified in the study of matrix inverses, a key concept for cryptography, solving systems of linear equations, and optimization, among other fields. Recent literature emphasizes the importance of authentic, contextualized experiences to

improve understanding and motivation [2]. Related work also reports that AI-driven contextualization can increase students' motivation when mathematical tasks are situated in a discipline-relevant setting [3]. This article contributes: (1) the design of a binational COIL task grounded in cryptography to situate matrix inversion within an end-to-end decryption workflow; (2) qualitative evidence of procedural autonomy and indications of conceptual understanding of invertibility; and (3) a pilot rubric (RCCI) to extract post hoc evidence of conceptual understanding of invertibility from student artifacts (reported here for indicators I1–I2). We do not report objective pre/post measures of learning; therefore, the findings are interpreted as exploratory, and we outline next steps to strengthen evaluation.

In this study, we use the Collaborative Online International Learning (COIL) approach, understood as the collaborative design of online learning experiences between classrooms in different countries to achieve disciplinary and intercultural objectives. Unlike simple virtual exchange, COIL emphasizes authentic tasks, faculty reciprocity, and assessment of collaboration [7], [8]. In our case, COIL is operationalized through mixed Mexico–Chile teams that solve a cryptography challenge and document their learning.

This study aims to address the gap in applied linear algebra within the Latin American context, specifically through a COIL collaboration between Mexico and Chile. By integrating matrix cryptography and SDG 4, the research explores how technical competencies can be harmonized with social responsibility. This approach not only enhances mathematical proficiency but also fosters a sense of global citizenship among engineering students in the region.

II. RELATED WORK

A. Collaborative Online International Learning (COIL) framework

COIL is a pedagogical approach that connects students and instructors from different countries to complete shared, collaborative tasks mediated by digital technologies, with explicit goals for intercultural competence and disciplinary learning. Although it is often grouped under the umbrella of virtual exchange, COIL emphasizes co-designed, reciprocal projects with assessment of collaboration rather than passive exposure [7]–[8]. Recent studies report positive effects of COIL on intercultural dispositions and teamwork skills and suggest domain-specific gains when tasks are authentic and loosely

structured. Our design leverages these features by pairing Mexico–Chile teams on a discipline-anchored cryptography challenge, making intercultural communication consequential for solving the task. Rubin's evolutionary framework [18] for embedding COIL highlights the importance of faculty co-design and reciprocal assessment—principles we operationalize through joint rubric development and synchronized delivery.

B. Contextualized Linear Algebra Through Cryptography and SDG 4

Prior COIL research has focused mainly on intercultural outcomes; fewer studies report disciplinary learning in mathematics through explicit algorithmic artifacts such as matrix inversion within a cryptographic workflow. By situating a linear-algebra topic (matrix inversion) within a real cryptographic workflow, we address this gap and examine the technical, pedagogical (SDG 4), and emotional–collaborative dimensions together.

In parallel, integrating SDG 4 (Quality Education) into curricular design has gained relevance. The United Nations General Assembly establishes SDG 4 targets such as equitable access, teacher training, and measurable learning outcomes as part of the 2030 Agenda [11]. By incorporating SDG 4 content into a mathematics task, this study adds disciplinary relevance and engages students in reflecting on equity and the feasibility of local actions. Thus, mathematics learning is positioned as a gateway to broader social awareness.

C. Methodological Framing for Trustworthy Thematic Analysis

Methodologically, ensuring rigor in qualitative research remains crucial. Practical guidance for trustworthy thematic analysis emphasizes transparency, traceability, triangulation, and a documented analytic process [12]–[14]. In this study, we follow these principles through an inductive codebook thematic analysis supported by investigator triangulation on a subset of the corpus, an audit trail (codebook versions and memos), and data triangulation across reports, presentations, and reflections.

D. Theoretical Framing: Active Learning and Gameful Design

The intervention follows an active-learning and constructivist logic: students build meaning by solving a consequential problem, negotiating strategies, and validating results rather than reproducing isolated procedures. Evidence in STEM education associates active learning with improved performance compared to traditional lecturing [21]. Additionally, framing inversion as a cryptography “challenge” introduces gamification elements (clear goal, feedback through verification, and a meaningful narrative), consistent with

definitions of gamification as the use of game-design elements in non-game contexts [22], [23].

III. METHODS

A. Design

We conducted a qualitative case study embedded within a classroom intervention based on COIL and matrix cryptography. Data sources included team reports, recorded presentations (automatically transcribed), and individual reflections. We employed an inductive codebook thematic analysis, using Atlas.ti for data management and visualization [12]. A qualitative approach is appropriate for capturing cognitive, affective, and sociocultural aspects of learning in context [15], and the authentic framing via SDG 4 (Quality Education) connects disciplinary content with real-world concerns [17].

B. Participants

Forty undergraduates from two universities (Mexico and Chile), enrolled in a matrix modeling course during Spring 2025, worked in mixed binational teams. The intervention took place between February and June 2025, with data collection completed by late June 2025.

C. Ethical Considerations

This study followed the ethical protocols of the participating institutions. All participants provided informed consent, and data were anonymized (e.g., T01-TEC, T01-UDD) to ensure confidentiality. Participation was voluntary, with no impact on academic grading for those choosing not to participate.

D. RCCI Instrument Status

The Rubric for Conceptual Understanding of Invertibility (RCCI) used in this study is presented in its pilot validation phase. It was developed to complement traditional procedural assessments by capturing the logical depth of student justifications regarding matrix properties

E. Application Context / Instruments

The interdisciplinary challenge integrated matrix inversion and multiplication to decrypt a 10×10 message whose content referenced **SDG 4 (Quality Education)**. Tools included Python, MATLAB, and Excel. Design rationale for matrix size: We selected a 10×10 matrix after pilot testing with 3×3 matrices in Fall 2024 ($n=12$ students, not reported here). The smaller matrices allowed trivial manual calculation, required minimal tool engagement, and led to individual rather than collaborative work. A 10×10 matrix necessitates computational tools (hand calculation is impractical within the allotted 90-minute session), introduces realistic complexity analogous to applied

cryptography, and requires team division of labor (e.g., one member handling matrix input, another verifying determinant, a third executing inversion). This size balances practicality with an authentic computational challenge typical of applied cryptography [6].

The activity unfolded in three steps over two weeks:

Step 1 (Decryption, ~90 minutes in-class). Teams recovered the plaintext by applying matrix inversion and matrix multiplication in a supervised computer lab session, and cross-validated results across tools.

Step 2 (Evidence-Action Memo, ~60 minutes out-of-class). Teams wrote a 200–300-word memo identifying SDG 4 targets, obstacles (e.g., equity, teacher preparation, access), and two feasible on-campus actions, each with an indicator. This step was completed asynchronously via shared documents.

Step 3 (Guided Reflection, ~30 minutes out-of-class). Each student submitted approximately 150 words on how linear algebra and computational tools shaped their understanding of SDG 4 and their future engineering practice. Reflections were submitted individually through the learning management system.

Assessment. The decryption task and the Evidence–Action Memo were implemented as formative learning activities. The Evidence–Action Memo rubric (0–2) evaluated team products on (i) disciplinary accuracy, (ii) linkage to SDG 4, and (iii) feasibility of actions; these scores were used for research reporting (team level), not as a high-stakes summative exam. Individual reflections were not survey-based; they were open-ended texts analyzed qualitatively. Separately, RCCI was applied retrospectively to team artifacts to extract evidence of conceptual understanding of invertibility.

F. Data and Analysis

We analyzed (1) team reports, (2) presentation transcripts, and (3) individual reflections. Atlas.ti supported open coding, code clustering, and network visualizations. Two analysts double-coded ~25% of the corpus, negotiated a master codebook (definitions and inclusion/exclusion criteria), and iteratively refined themes through memoing and peer debriefing. Following standard phases of thematic analysis—familiarization, initial coding, theme development/review, definition/naming, and reporting [12]—we triangulated across sources and maintained an audit trail (versions of the master codebook and memos) to strengthen trustworthiness. Quotations are labeled by team (T01–T09) and, when illustrative, with institution suffixes (–TEC/–UDD) to preserve anonymity and show the diversity of voices. Table I summarizes data sources and processing tools.

TABLE I
DATA SOURCES AND ANALYSIS TOOLS

Data Source	Purpose	Tool / Processing Method
Team Reports	Document the technical decryption process and reflections on SDG 4 meaning	Written reports analyzed with Atlas.ti
Final Presentations	Capture discourse and emotional aspects not present in written reports	Recorded; automatic transcription
Individual Reflections	Collect personal perspectives on mathematics, tools, and the relevance of SDG 4.	Texts coded in Atlas.ti

About Atlas.ti. We used Atlas.ti as a qualitative management platform to organize the corpus, segment sources into meaningful fragments, and link them to analytic codes. The software facilitated typical thematic-analysis operations: code-based search and retrieval, memoing, co-occurrence analysis, and network visualizations to explore relationships among categories. What do we mean by “codes”? In this study, a code is an analytic label assigned to one or more data segments (words, sentences, or paragraphs) that share a common meaning. Codes were generated inductively during iterative reading and refined in a master codebook (definitions, inclusion/exclusion criteria, and examples). Examples include “Use of computational tools,” “Understanding of cryptography,” “Application of linear algebra in real contexts,” “Education as a human right,” and “Intercultural collaborative work.” Codes were then grouped into three themes: technical, pedagogical, and emotional–collaborative.

All quotations were originally produced in Spanish and were translated into English by the authors for reporting purposes.

G. Analysis Procedure

To address the need for more direct evidence of conceptual understanding, we designed the Rubric for Conceptual Understanding of Invertibility (RCCI) and propose its retrospective application to the artifacts already collected (reports and presentations). The RCCI comprises four indicators scored on a 0–2 scale (0 = absent/incorrect; 1 = partial/applied without sufficient justification; 2 = correct with explicit, transferable justification):

I1. Invertibility conditions and justification. Identifies and justifies conditions (e.g., $\det(A) \neq 0$; full rank; singular/non-singular) and explains why.

I2. Diagnosis and error correction. Detects causes of non-invertibility or procedural failure (e.g., zero determinant, misdimensioning, invalid key) and proposes corrections.

I3. Method selection and justification. Compares or selects methods (Gauss–Jordan, adjugated/cofactors, LU, or computational tools) with explicit criteria (complexity, stability, verifiability).

14. Transfer/generalization. Extends reasoning to variants (other sizes, matrices with special properties, numerical stability considerations) or explains procedural limits.

Procedure. Two raters independently score the artifacts (unit of analysis = team) and resolve discrepancies by consensus. We report the percentage of teams with full evidence and we include short excerpts to illustrate typical justifications. Table II documents operational definitions and performance anchors.

TABLE II
RUBRIC FOR CONCEPTUAL UNDERSTANDING OF
INVERTIBILITY (RCCI)

Indicator	0 (absent/ incorrect)	1 (partial)	2 (complete/ justified)
11. Invertibility conditions and justification	Does not mention conditions; conceptual errors (e.g., confuses determinant with trace). Example: "We inverted the matrix A."	Mentions a condition (e.g., $\det \neq 0$) without explanation. Example: "We checked $\det \neq 0$."	Identifies conditions and justifies them using properties (rank, non-singularity, uniqueness). Example: "Since $\det(A) = -12 \neq 0$, the matrix is full rank and A^{-1} exists and is unique."
12. Diagnosis and error correction	Does not identify the cause of error or attributes it vaguely. Example: "It didn't work."	Detects the cause but does not explain the correction. Example: "We got zero determinant so we tried again."	Detects the cause and corrects it with explanation. Example: "The zero determinant indicated a singular key matrix. We verified entries, found a transposition error in row 3, and recomputed."
13. Method selection and justification	Applies a method without criteria; arbitrary choice. Example: "We used Python because it's easier."	Chooses/compares methods with an implicit criterion. Example: "Excel was faster for small tests."	Chooses/compares using explicit criteria (stability, complexity, verifiability). Example: "We used Gauss-Jordan in Python for automation and verified with Excel's MINVERSE to cross-check numerical stability."
14. Transfer/generalization	No transfer; does not discuss limits.	Mentions variants without analysis.	Discusses variants/limits and generalizes reasoning.

	Example: No mention of other scenarios.	Example: "This might work for bigger matrices."	Example: "For larger matrices (50×50), iterative methods like LU decomposition would be more stable than direct inversion due to floating-point accumulation."
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H. Trustworthiness and Validity Measures

To enhance the credibility and dependability of our qualitative findings, we implemented several trustworthiness strategies aligned with established criteria for rigorous qualitative research [12], [14]:

Investigator triangulation: Two analysts independently coded approximately 25% of the corpus (n=3 team reports, n=90 reflection segments). Initial agreement reached 82% at the code level. Discrepancies were resolved through consensus discussion, resulting in the refinement of 12 code definitions in the master codebook. The remaining 75% was coded by the primary analyst, with regular peer debriefing sessions to discuss emergent patterns and ambiguous segments.

Data triangulation: We analyzed three distinct data sources (team reports, presentation transcripts, individual reflections) to capture technical, discursive, and personal dimensions of the experience. Convergence across sources strengthened confidence in thematic saturation.

Audit trail: We maintained four versions of the master codebook (dated March 15, April 10, May 5, and June 8, 2025), documenting code additions, mergers, and definition changes. Atlas.ti project files and analytic memos are archived for verification.

Thick description: We provide representative quotations and contextual details to enable readers to assess transferability to their own settings [15].

Reflexivity: As instructors of the participating courses, we acknowledge potential confirmation bias toward positive outcomes. To mitigate this, presentation transcripts were automatically generated (minimizing selective recording), and the second coder was not involved in course delivery. We also sought disconfirming evidence during coding, specifically marking instances of student confusion, frustration, or disconnection from SDG 4 content. Thematic saturation: Novel codes ceased emerging after analysis of the seventh team (T07), with the final two teams primarily reinforcing existing categories. This suggests adequate sample size for the exploratory scope of this study.

These measures do not eliminate interpretive subjectivity inherent to qualitative research but provide transparency and traceability that allow readers to evaluate the trustworthiness of our findings.

IV. RESULTS

The thematic network (Fig. 1) includes 21 core codes organized into three interrelated dimensions: technical, pedagogical, and emotional–collaborative.

A. Technical-Pedagogical Intersection:

Data show that technological mastery served as a vehicle for social reflection. The code "Use of computational tools (Python, MATLAB, Excel)" showed a direct relationship with the view of "Education as a human right." Furthermore, "Understanding cryptography" allowed students to achieve a clear "Connection with SDG 4." A key finding is that "Overcoming technical errors" during the 10×10 matrix inversion was linked by students to the "Social impact of mathematical knowledge". One student described cross-tool verification as follows: *"To recover the original matrix, we did the activity twice to check the results—first using Matrix Calculator and then using Python with NumPy."* (T02-TEC, translated from Spanish). Another student highlighted a common technical adjustment during decryption: *"We rounded the results and converted them to integers, because decimals were causing errors."* (T05-UDD, translated from Spanish).

B. Pedagogical Dimension and Critical Reflection:

The code "Critical reflection on educational equity" emerged from the "Application of linear algebra in real contexts." Students reported "Significant learning" by realizing that mathematical tools can support efforts to address structural gaps. A participant linked the decrypted message to education as a core value: *"Decrypting with inverse matrices helped us discover a message aligned with values like education, and it showed that technology can protect messages with purpose."* (T07-TEC, translated from Spanish). A second reflection emphasized perceived relevance and interest: *"It was different from what we usually do; applying matrices in a more real context made it more interesting, and it made us want to keep exploring topics like this."* (T07-UDD, translated from Spanish). All teams (9/9) proposed concrete on-campus actions with indicators; these products were evaluated with the Evidence–Action rubric (0–2) described in Methods (disciplinary accuracy, linkage to SDG 4, and feasibility). Table IV summarizes the Evidence–Action Memo rubric results (unit of analysis = team, $n = 9$): 44% (4/9) achieved full evidence in disciplinary accuracy, 67% (6/9) in linkage to SDG 4, and 56% (5/9) in feasibility of proposed actions. Separately, to provide post hoc evidence of conceptual understanding of invertibility,

we applied the RCCI to team artifacts: 78% of teams provided complete justification of invertibility conditions ($I1 = 2$), and 67% demonstrated systematic error diagnosis and correction ($I2 = 2$). Table III reports RCCI results at the team level (unit of analysis = team, $n = 9$). For each indicator, we summarize the proportion of teams that provided full evidence (score = 2) in their artifacts. Thus, 78% (7/9) of teams met the "complete" criterion for $I1$ (invertibility conditions and justification), and 67% (6/9) met the "complete" criterion for $I2$ (diagnosis and error correction). Indicators $I3$ and $I4$ were not assessed in this iteration and are planned for scoring in the next cycle.

TABLE III
RCCI RESULTS (UNIT OF ANALYSIS = TEAM, $n = 9$)

Indicator	Full evidence (2)	Brief interpretation
I1. Invertibility conditions and justification	78% (7/9)	Complete justification (e.g., $\det(A) \neq 0$ with explanation)
I2. Diagnosis and error correction	67% (6/9)	Systematic identification and correction of errors
I3. Method selection and justification	Not assessed in this iteration	Planned scoring in the next iteration
I4. Transfer / generalization	Not assessed in this iteration	Planned scoring in the next iteration

TABLE IV
EVIDENCE–ACTION MEMO RUBRIC RESULTS (UNIT OF ANALYSIS = TEAM, $n = 9$)

Criterion (0–2)	Full evidence (2)	Brief interpretation
Disciplinary accuracy	44% (4/9)	Correct mathematical/technical rationale supporting claims
Linkage to SDG 4	67% (6/9)	Explicit alignment with SDG 4 targets and barriers
Feasibility of proposed actions	56% (5/9)	Actions are realistic for an on-campus context and include an indicator

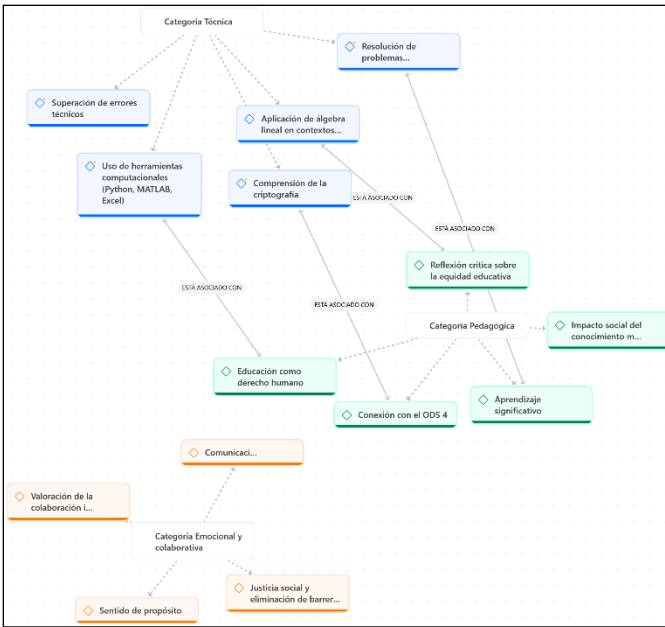


Fig. 1. Network map of co-occurring codes: Interrelation between technical, pedagogical, and emotional dimensions.

C. Emotional and Collaborative Dimension:

This dimension was vital for project resilience. "Valuing international collaboration" and "Intercultural communication" mitigated frustration during the technical challenge. One student described coordination difficulties and team convergence: *"At the beginning it was hard to agree on how to document everything, and coding was complex because everyone interpreted it differently, but in the end we organized well as a team."* (T03-TEC, translated from Spanish). Another student emphasized collaboration and resilience: *"Decoding the matrix reinforced the theory, but it also showed the value of collaborative work regardless of distance and resilience when facing technical challenges."* (T03-UDD, translated from Spanish). Across reflections and presentations, students emphasized intrinsic motivation, a sense of purpose, and joint problem solving as drivers of persistence. The category "Social justice and elimination of structural barriers" operated as a shared narrative that helped binational teams connect the technical task with SDG 4 concerns.

V. DISCUSSION

A. From Technique to Social Awareness

Unlike traditional teaching where mathematical work may stop at computing a determinant, this model encourages students to connect technique with meaning. The results suggest that when students complete an end-to-end decryption workflow, they feel more confident to discuss broader issues such as educational equity in a concrete, indicator-driven way. This aligns with LACCEI's interest in forming engineers with

strong technical competence and ethical commitment. From a learning-theory perspective, the task can be interpreted as active learning grounded in constructivist assumptions: students construct meaning by engaging in an authentic, goal-directed problem, negotiating strategies in teams, and using verification checks (cross-tool validation and successful decryption) as feedback loops rather than reproducing isolated procedures. Meta-analytic evidence in STEM indicates that active learning improves student performance relative to traditional lecturing [21]. Moreover, presenting inversion as a cryptography challenge adds a gameful framing—an explicit goal, immediate feedback through verification, and a narrative payoff—consistent with the gamification literature's emphasis on using selected game-design elements to support engagement in non-game learning contexts [22], [23].

B. The Impact of Binational Collaboration

The emotional dimension reveals that the Mexico-Chile collaboration was an experience of "Intercultural communication" that generated a "Sense of purpose." "Social justice" shifted from an abstract concept to a shared goal, suggesting that COIL environments are ideal for humanizing basic sciences.

C. Scalability and Regional Relevance

The use of codes like "Its application ranges from banking transactions." shows that students understand engineering's versatility. Although the sample size is a limitation, the strong links between technique and pedagogy suggest this model may be scalable across Latin America, provided that instructors ensure tool access, sufficient lab time, and structured collaboration scripts for mixed teams. Scaling to larger cohorts requires attention to infrastructure and instructor workload. Technological constraints include reliable computer-lab access, software availability (or a common open-source stack), and enough time for cross-tool verification. Workload increases mainly in (i) binational coordination (scheduling and shared materials), (ii) troubleshooting during the lab session, and (iii) evaluating team artifacts. To mitigate this, we recommend standardized templates for matrix input/output, pre-configured verification scripts (e.g., determinant and inverse checks), and distributing facilitation roles (one instructor + teaching assistants or peer facilitators per lab section).

D. Synthesis of Findings

- Working with invertible matrices in a complete decryption pipeline clarified the "why" of the inverse, not only the how.
- Reports and reflections show procedural autonomy—fixing key-matrix issues, checking determinants, and scripting workflows across tools.

- The COIL experience consistently elicited intercultural communication and joint problem solving in binational teams.
- Students linked decrypted content to specific SDG 4 targets (e.g., teacher training, equity of access) and proposed feasible actions with indicators.
- Purpose, challenge, and international teamwork emerged as central motivational drivers that renewed confidence in mathematics.

E. Limitations

This study has several limitations. First, its scope spans only two institutions and a single course iteration, which constrains external validity. Second, much of the evidence derives from self-reported reflections that are susceptible to social desirability and recall bias. Third, the design lacks pre/post-performance measures, so we cannot estimate objective learning gains beyond perceived outcomes. In the next iteration, we will add a short pre/post performance task aligned with invertibility reasoning (not only procedural inversion) and report average gains and an effect size (e.g., Cohen's d) to strengthen empirical validation. Finally, findings stem from interpretivist analysis and are context-dependent; we mitigated this through triangulation across team reports, presentations, and individual reflections, a documented audit trail, and double-coding with negotiated agreement on a subset of data, yet residual subjectivity remains. These boundaries of inference motivate multi-site, longitudinal replications with mixed-methods assessment. We also did not measure knowledge retention; a delayed post-task (e.g., 4–6 weeks later) is planned to examine whether conceptual justifications of invertibility persist beyond the activity.

VI. CONCLUSIONS

This study implemented a binational COIL cryptography task to teach matrix inversion through an end-to-end decryption workflow. Students worked in mixed teams ($n = 9$) using Python, MATLAB, and Excel to decrypt a message encoded with a 10×10 key matrix, and then produced an evidence–action memo connecting the decrypted message to SDG 4 with feasible on-campus actions and indicators.

Analysis of team reports, recorded presentations, and individual reflections identified three recurring dimensions across the dataset: (i) technical learning (cross-tool verification and procedural autonomy), (ii) pedagogical and contextual meaning (explicit linkage to SDG 4 and equity-oriented reflection), and (iii) emotional–collaborative factors (intercultural communication and persistence during troubleshooting).

As post hoc evidence of conceptual understanding, we applied the RCCI to team artifacts. Seven of nine teams (78%) provided

complete justification of invertibility conditions ($I1 = 2$), and six of nine teams (67%) demonstrated systematic diagnosis and correction of errors ($I2 = 2$). Because no objective pre/post measures were collected, these results should be interpreted as exploratory evidence derived from authentic project artifacts. Future work will add performance-based pre/post tasks, report all RCCI indicators ($I1$ – $I4$), and expand the sample across additional course sections to strengthen inferences about learning outcomes.

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