ISBN: 978-628-95207-4-3. ISSN: 2414-6390. Digital Object Identifier: 10.18687/LACCEI2023.1.1.754

Generalized Kutta Joukowski theorem adaptation to classical Horseshoe Vortex Method for modeling multi-lifting surfaces interaction: Application to UAV's wing layout design.

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Abstract– In this study, an adaptation of the classical Horseshoe Vortex Method is performed to model the interaction of two or more lifting surfaces employing the generalized Kutta Joukowski theorem, which allows computing the lift produced by both inside and outside vortices. The resulting model was validated against wind tunnel tests for a canard airplane and showed good results for lift and induced drag predictions, especially before the region of flow detachment. The panel method obtained is useful for computing preliminary aerodynamic characteristics of an aircraft with a non-conventional wing configuration considering the respective model limitations and assumptions. Thus, a case study is developed comparing the aerodynamic performance of the conventional and tandem wing layout configurations for a small fixed-wing UAV of 10 Kg of maximum take-off weight. Among the most important results, it was found that the tandem-configuration aircraft showed a reduction of 12% of the lift generated compared with the lifting-line theory prediction when forward and rear wings aerodynamic interaction was considered. Also, for the tandem configuration aircraft, it was encountered to have a larger range of applicability due to its smaller size; however, the conventional one showed less interference in the airflow between the wing and horizontal stabilizer, consequently, better aerodynamic efficiency.
Keywords - Horseshoe Vortex Method, UAVs, wing design, aerodynamics, Kutta Joukowski theorem

I. INTRODUCTION

During the preliminary design process, the layout of the aerodynamic surfaces is specified to establish the main characteristics and limitations of the aircraft's performance during flight, according to the parameters of a selected mission. Hence, to choose a wing layout configuration that fulfills the design requirements, different methods have been developed and used over the years, which may include simplified mathematical models or tests in wind tunnels.

Broadly speaking, to predict the aerodynamic performance of an aircraft, there are three main groups: analytical solutions, numerical solutions, and wind tunnel studies.

The first group contains the analytical solutions developed throughout the history of aerodynamics, where the development of Prandtl's lifting-line theory provided the first analytical method that accurately predicted the distribution of lift across the wing surface [1] by sectioning the wing and applying the circulation concept of the Kutta-Joukowski theorem, to a circulation distribution along the section [2]. However, conventional Lifting-line theory applies only to a single lifting surface with neither sweep angle nor dihedral [2]. On the other hand, there are Computational Fluid Dynamics (CFD) methods for modeling wind flow, which have become, in recent years, a reasonably accurate and useful tool to know the behavior of flow through the wing [3]. However, this method implies an in-depth knowledge of the model to be developed with a relevant complexity in geometry meshing, especially when it has large dimensions or sudden changes in geometry [4] since the interpretation of the results is a critical point. It is also not quite accessible due to the considerable computational cost involved.

Similarly, physical wind tunnel experimentation may be another of the most accurate ways to measure aerodynamic forces on a three-dimensional wing surface but, as with CFD, this is expensive and even more limited to the general public. Thus, simplified numerical methods tend to be the midpoint between costs and precision in predictions like the Horseshoe Vortex Method (HVM), which is a simple panel method of very small computational cost, and that allows the inclusion of complex geometric parameters [5]. This is a simplified representation of the vortex system of a wing, where the vorticity of this can be presented as a vortex of the constant circulation that travels through it, and two vortices at the wingtips of the opposite sign to the direction of advance of the aircraft, which appear immediately the wing begin to move through the flow [6]. In this hypothesis, the viscosity is dismissed but is implicitly incorporated into the Kutta condition; therefore, the lift calculated from the principle of the Kutta-Joukowski theorem is considerably precise even for a real viscous flow, as long as it is constant and it does not separate from the wing [7].

Nevertheless, these mathematical approaches have been developed primarily for aircraft with a single fixed-wing surface, leaving out atypical multi-surface configurations such

Digital Object Identifier: (only for full papers, inserted by LACCEI).
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as the tandem, thus limiting conventional mathematical models. For these reasons, this document aims to present a modification of the Horseshoe Vortex Method (HVM) by applying the generalized Kutta-Joukowski theorem, obtaining a numerical model for multiple lifting surfaces that considers the interaction of the flow between the wings, which could be applied to a wide range of wing layout configurations. This model is successfully validated and shows its applicability in this paper through a comparative aerodynamic study between two wing configurations: conventional and tandem, for a fixed-wing aircraft that will have the same maximum gross takeoff weight as the starting point for comparison.

Therefore, this paper seeks to provide tools that are accessible to respond to the challenge currently involved in the design of UAVs, rescuing the tandem as a viable configuration since it was considered a winning solution from the first days when the flights began, manned or not. Although, throughout the 20th century, some engineers and designers focused on studying and experimenting with aircraft with this type of configuration [8]. After a few years of experiments, the conventional configuration was established as an effective and reliable solution for manned aircraft, remaining to this day as the best-known and most used [9].

However, given the imminent current growth of the UAV industry and the design versatility they present within a wide variety of missions [10], traditional sizing methods can be reformulated, modifying the theories typically applied to conventional concepts of design. This implies that it is possible to resort to innovative and more practical designs than the pre-established ones, which offers the possibility of studying different configurations of wing surfaces according to the initially required operating parameters since the aerodynamic characteristics and performance limitations of a fixed-wing aircraft are mainly determined by the geometric configuration of the wing it has [11].

This article is presented in the following order: in section II, a description of the model implemented in the study is presented, both in the sizing paragraph and in the implemented calculation model, later the results obtained in section III are shown with their respective analysis, and finally, the conclusions are developed in front of the obtained results and the opportunities for the continuation of the investigation.

II. METHODOLOGY

In this study, a numerical method is presented based on an adaptation of the classical HVM with the generalized Kutta-Joukowski theorem, which calculates the aerodynamic properties of a multiple-lifting-surface system, such as the induced drag, and the lift force, considering the interaction of the airflow through multiple lifting surfaces. Additionally, the methodology is engaged with an empirical approach to calculate the parasite drag and compare the conventional and tandem configurations to determine the comparative advantages and disadvantages of main aerodynamic characteristics. The study case aims to highlight the assets that may be needed in several different UAV applications.

A. *Generalized Horseshoe Vortex Method*

HVM is a simple panel method that corresponds to a lifting-line numerical solution and can be extended easily to include complex geometrical effects [5], providing a suitable solution to base the modeling of the aerodynamic characteristics as a first stage during the design layout. However, the method must be modified to obtain more accurate aerodynamic properties of multiple lifting surfaces interacting and influencing each other, which can be the case of multiplane, tandem, canard, and other wing configurations.

The HVM can be used to compute the lift and induced drag of a wing with acceptable accuracy if its limitations are known, namely, [5]:

- The small disturbance assumption: limits to soft changes in potential flow.
- Thin airfoil simplification: admits values of relative thickness less than 0.15.
- Large aspect ratios (AR) requirements allow AR larger than 4.
- Attached fluid condition: limits to α values where low detachment is guaranteed.

To model the aerodynamics of a three-dimensional wing with HVM, the wing platform is discretized into N sections through the span formed by a straight bound vortex segment that represents the lifting properties of the wing, and two semi-infinite trailing vortex lines that model the wake, obtaining a horseshoe element [5].

Each horseshoe element influences a control point located in the middle of the wake and at $3/4$ of the chord behind the leading edge. The Biot-Savart law is implemented by applying the no-slip condition to calculate the induced velocity of these elements in the control points. The governing equation can be expressed in (1):

$$A_{ij}\Gamma = -(U_\infty, V_\infty, W_\infty)\hat{n}_i \qquad (1)$$

Where A_{ij} is the matrix with influence coefficients, Γ is the vector with the value of circulation in each control point $\hat{n}_i$ is a matrix with normal vectors at each pane U_∞, V_∞ and W_∞ are the components of the free stream speed in the coordinate axis. If the circulation is obtained from Eq. (1), the lift can be obtained by applying Kutta Joukowski theorem [5] (2):

$$\Delta L_j = \rho V_\infty \Gamma_j \Delta y_j \qquad (2)$$

Where ΔL_j is the lift of each panel, V_∞ is the speed of the free stream flow, and Δy_j is the width of each Horseshoe element. Note at this point that this equation only considers one source of circulation. Therefore, Eq. (2) needs a reformulation to adapt the model to consider multiple sources of circulation, which is presented by Bai and Wu [7] and can be written as (3):

$$\Delta L_j = \rho(V_\infty + u_s)\Gamma_j \Delta y_j \quad (3)$$

Where u_s is the velocity at the section induced by an outside vortex Eq. (3) shows that the effect of external sources of circulation is considered in the velocity component of Eq. (2). In Eq. (1), the velocity induced by the vortex segments in the control points are contained in the matrix A_{ij}, which implies that the equation should be modified to consider the velocity induced by an external vortex, obtaining a governing equation for multi-lifting surfaces. For simplicity, named Generalized Horseshoe Vortex Method (GHVM) (4):

$$\left(A_{i1j1} + A_{i1j2} + \cdots A_{i1jN}\right)\Gamma_1 = -(U_\infty, V_\infty, W_\infty)\hat{n}_i \; (4)$$

Where A_{i1j1} represents the matrix of influence coefficients over the control points of the surface number one due to bound vortices and trailing vortices of itself, and A_{i1j2} contains the influence coefficients over the control points of the surface number one, due to the vortices that represent the surface number two, with N as the number of interacting lifting-surfaces. The circulation Γ_1 calculated with Eq. 4 corresponds to one wing, and the other lifting sources can be modeled similarly. A scheme of the application of this model in a tandem wing arrangement is shown in Fig. 1.

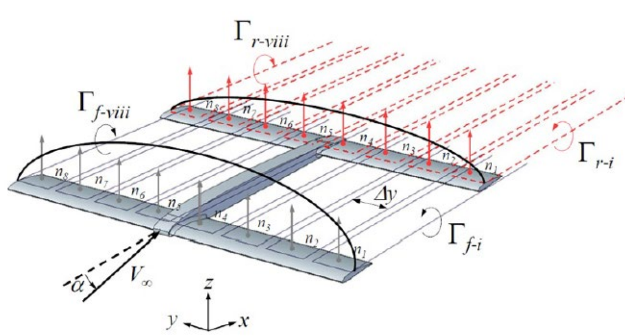


Fig. 1 Scheme of the discretization and application of the GHVM for a biplane case, with horseshoe elements colored in gray for the forward wing (sub-index f) and in red for the rear wing (sub-index r).

Additionally, in the classical HVM theory, the induced drag can be computed by accounting for the downwash induced by the trailing vortices as (5):

$$D_{ind} = -\frac{1}{U_\infty}\sum_{i=1}^{N} \Delta L_i \alpha_i^{ind} \quad (5)$$

Where the angle of attack induced α_i for each panel depends on the *z* axis component of the induced velocity by the wake, as follows (7):

$$\alpha_i^{ind} = -\frac{1}{U_\infty}\sum_{i=1}^{N} \Gamma_j \widetilde{\omega}_{ij}^{wake} \quad (7)$$

Now, when the interaction between more lifting surfaces must be accounted for, the α_i for each wing panel depends not only on the velocity induced by the wake of that surface but also on the bounded and trailing vortices of the other interacting surfaces, thus:

$$\alpha_{i,1}^{ind} = \sum_{i=1}^{N} \Gamma_j\left(\widetilde{\omega}_{ij1}^{wake} + \widetilde{\omega}_{ij2} + \widetilde{\omega}_{ijN}\right)$$

Note that the GHVM can be used to compute aerodynamic characteristics for a big variety of wing configurations like biplane, tandem, and even conventional, considering in the last one the horizontal stabilizer as the rear lifting surface.
To solve this numerical method, a computational implementation in Matlab programming language is developed.

B. Parasite Drag Approximation

The GHVM adaptation developed cannot account for the drag effects produced by the shape and the friction between the bodies and the surrounding airflow, which is an important component to consider in the aerodynamic properties of an aircraft, as the basis of the calculations are the inviscid flow assumptions. Nevertheless, there are simplified methods that allow getting an approximation of the parasite drag of wings, such as the equivalent skin friction method [12], which is based on the calculation of the friction coefficient of an equivalent flat plate.

For laminar flow, that is for *Re < 500:000*, the friction coefficient can be approximated as [13] (8):

$$C_f = \frac{1{,}328}{\sqrt{Re}} \; (8)$$

With this coefficient, an approximation of the parasite drag of the wings, without taking into account minor components of the drag as produced by leakages and protuberances, can be obtained as [12](9):

$$C_{D0} = \frac{C_f \times FF \times Q \times S_{wet}}{S_{ref}} \quad (9)$$

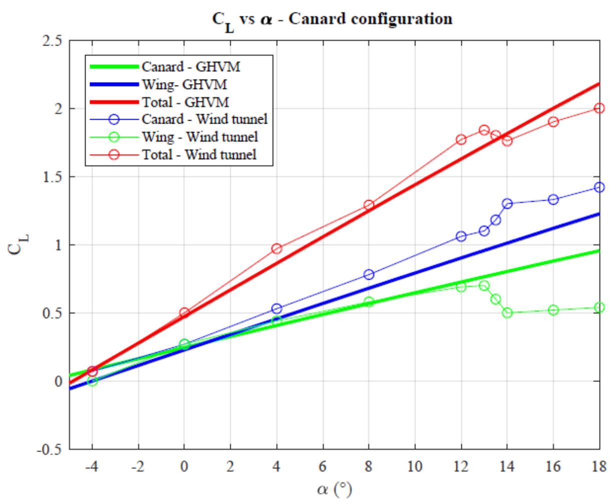
FF is the form factor that depends on the airfoil maximum thickness, its location, the wing sweep angle, and the interference factor *Q*, which quantifies the drag augmentation by the mutual interaction of the components. In this manner, a total drag approximation can be obtained as follow (10):

$$C_D = C_{Dind} + C_{D0} \quad (10)$$

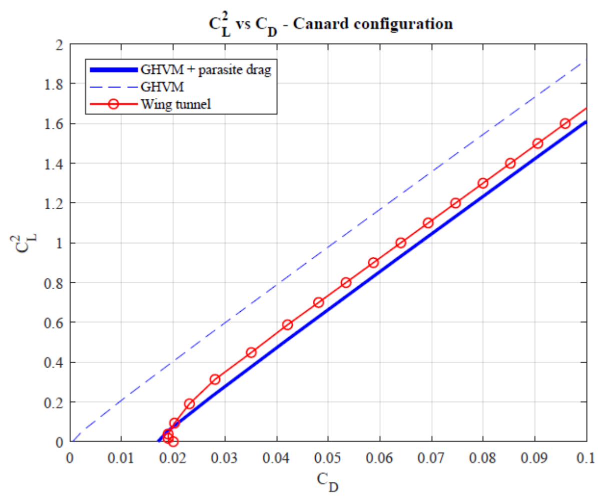
Where the C_{Dind} is calculated with the GHVM adaptation and the C_{D0} is obtained from Eq. 9.

C. Numerical Method Validation

To verify the validity of the model obtained in Eq. (4) and (10) and its implementation, the aircraft tested in a wind tunnel by Feistel, Corsiglia, and Levin [14] is introduced into the computational implementation of the model. The validation results are presented in Fig. 2, where the bold lines correspond to the calculations performed with GHVM for multi-lifting surfaces. At low angles of attack, there is a good match between numerical and experimental values, obtaining a relative error of 5,67% for the total lift slope, while the viscous effects and the detachment of the fluid increase the method's relative error at higher angles (after 13°). In terms of drag, the result of the slope of the curve *CL^2 vs. CD* presents a difference of 2,86% compared with the experimental results and an approximated difference of $3{,}5 \times 10^{-3}$ the C_{D0} value.



(a) Lift validation



(b) Induced drag validation

Fig. 2 Validation of the horseshoe vortex method for multi-lifting surfaces with wind tunnel measurements (a), and the result of drag coefficient (b)

The results for the induced drag presented in Figure 2 (b) in blue dashed lines are more accurate compared with the reported results for the classical Prandtl-Munk theory, which present a relative error of 25% approximately and are slightly better than the vortex lattice predictions, that present a relative error of 3,1%, compared with the experimental results also reported by Feistel et. Al [14]. Considering the parasite drag with the Eq. 9, the method presented in this paper allows obtaining an overall better approximation for the total drag characteristics for the canard airplane modeled than the numerical methods mentioned. Despite the difference in the lift values obtained, mainly at high α, the method allows obtaining a good approximation of the main aerodynamic properties before the angle of stall, in the range where usually the operational environment of the aircraft is placed. These results can be obtained at a relatively low cost compared to wind tunnel tests and other more sophisticated Computational Fluid Dynamics (CFD) software.

D. Case study

During the conceptual design of most fixed-wing aircraft, it can be required to establish the mission's requirements, propose several layouts of aircraft configurations, define wing parameters of the preliminary configuration, establish preliminary weight and CG, and estimate the drag. In low-cost projects, this conceptual design can be accomplished by applying the approximation of Lifting Line Theory (LLT) to obtain the wing parameters and calculate a preliminary CG; however, these results require a deeper analysis to obtain more accurate aerodynamic characteristics, which are obtained usually through CFD or wind-tunnel analysis.

Nowadays, several UAV applications require to design of the aircraft to fulfill a certain mission. For this reason, this case study aims to highlight the advantages and disadvantages of two wing configurations to detail the aerodynamic characteristics of the conventional and tandem configurations, which can be implemented in wing-fixed UAVs.

Since this case study compares two wing configurations completely different, it is necessary to set several characteristics as points of comparison for fair purposes. These criteria belong to the main part of every preliminary design process: the stability analysis, which describes the minimum requirement of the ability of the aircraft to maintain a stable and controlled flight. To obtain a naturally statically stable aircraft, the following aspects must be calculated or determined [15], the position of the center of gravity (CG), the airfoils of wings and stabilizers, and the dimensions and disposition of the wing surfaces. To obtain the latter parameter, the following points of comparison are established:

- A maximum gross takeoff weight of 10 kg for both configurations, which is a typical value for small

UAVs, according to the U.S. Department of Defense [16].

- The rectangular geometry is defined in all wing platforms for both configurations to standardize the aerodynamic performance.
- A gap, i.e. vertical separation, of half of the average chord between the two wings is established since it is a value usually used for biplanes to minimize the downwash effect in the rear surface [12].
- The airfoil for all wing platforms is a Cavini 15, except for the horizontal stabilizer of conventional UAV, which has a NACA 0015. This exception is because the conventional configuration frequently uses symmetrical airfoils on the rear surface [17].
- The CG position is slightly behind the aerodynamic center in conventional configuration [18], while in tandem, the CG is located well behind the forward surface [12].

As a first approximation, the classical LLT is used to obtain the approximated lift generated with the surfaces that promote the conditions of a longitudinal static stable system: when the pitching moment coefficient about the CG with zero lift is positive and the slope of the curve of pitching moment versus the absolute angle of attack is negative [15]. This procedure leads to obtaining the first value for the iterative process to dimension the aerodynamic surfaces of both configurations with the GHVM. The dimensions obtained to have the same gross take-off weight with the tandem and conventional configuration are presented in Table I.

TABLE I. Geometric characteristics for each configuration

	Tandem		Conventional	
	Front	Rear	Front	Rear
b [m]	2.10	2.10	3.71	1.64
c [m]	0.25	0.30	0.35	0.33
S [m^2]	0.50	0.60	1.35	0.54
α_{ind}	3.70	0.70	0.400	-0.890

III. RESULTS AND DISCUSSION

The definitive geometric characteristics computed for both configurations are schematized in Fig. 3. Compared with the LLT predictions, the dimensions are changed significantly when calculating the lift with the GHVM. This is caused by the lack of modeling of the flux interaction between forward and rear surfaces in the LLT calculations. Thus, applying the GHVM model, the weight that can be lifted by the tandem is reduced by 12% of the value previously calculated with LLT, while the conventional one augmented in 4% its lifting characteristics because of positive interaction. Therefore, the tandem span was increased by 10 cm for each wing, and the conventional wingspan was reduced by 14 cm compared with the dimensions predicted with LLT, encountering the defined weight of comparison of 10 Kg.

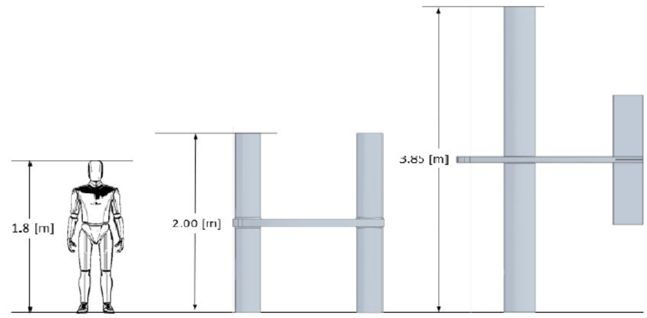


Fig. 3 Comparison of the UAVs sized for the same payload with different wing configurations.

Having the geometry of both configurations defined, the GHVM is applied to obtain the aerodynamic characteristics of interest. The stall behavior is obtained by calculating the C_L and its corresponding α at which the lift distribution along the span reaches the $C_{l,max}$ on any section. Table II shows the mentioned parameters, where it is important to note that the reference area to calculate the characteristics mentioned is the same to obtain a fair comparison, corresponding to the area of the main wing of the conventional configuration.

TABLE II. Summary of aerodynamic characteristics

Configuration	$C_{L,\alpha}$ [deg^{-1}]	$\alpha_{L=0}$	$C_{L,max}$ [deg]	α_{stall} [deg]
Conventional	0.1145	-4.99	1.225	7.06
Tandem	0.0688	-8.34	0.560	5.50

Table I shows that the conventional configuration has a total area equal to 1,89 m^2, while the tandem configuration has a total area of 1.1 m^2. Hence, the conventional configuration requires a larger area to provide the same total lift. This implies that the tandem configuration can provide greater lift with a smaller surface than the conventional one, which translates into beneficial effects in terms of structural weight reduction [8].

Nevertheless, Table II indicates that the conventional configuration has a $C_{L,max}$ and an α_{stall} greater than the tandem configuration. These characteristics, in addition to the angles of incidence indicated in Table I, imply a better behavior to stall and pitch stability for the conventional configuration.

However, the stall behavior of the tandem configuration has an advantage compared to the conventional one because the secondary wing can continue providing lift when the front wing stalls, leading to a recovery from that undesirable condition if the lifting surfaces are properly designed.

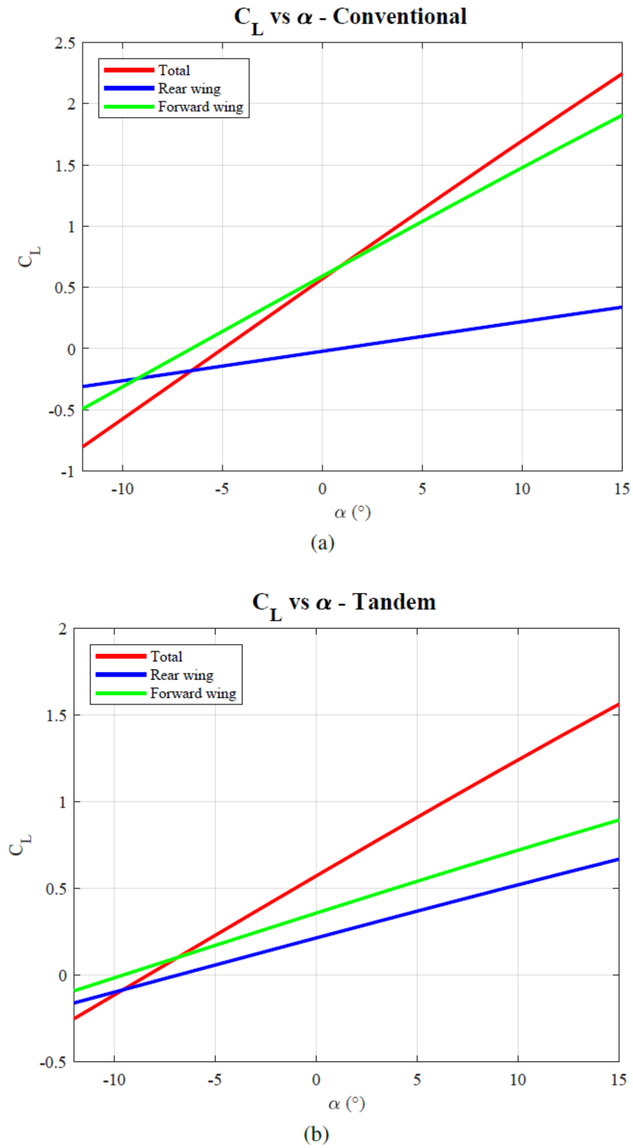


Fig. 4 Lift coefficient vs angle of attack for conventional (a) and tandem (b)

Fig. 4 shows that the front wing of the conventional configuration provides most of the lift, while in the tandem configuration, both the front and rear wings provide a considerable part of the total lift. Nonetheless, the conventional configuration has a range of operations in terms of angle of attack where the total lift is lower than the front wing lift because of the negative contribution of the horizontal tail required for stability until *alpha* = 1.10°, while this same behavior is obtained right after *alpha* = -6.8° in the tandem configuration. This latter performance shows that the tandem wings are more efficient in providing lift in a large range of α, as the lift produced by the rear wing is more exploited.

As explained in Eq. 10, the total drag is calculated as the sum of the induced drag and an approximation of the parasite drag. Figure 5 shows that the tandem configuration has a minimum C_D than the conventional configuration. However, this is true within negative angles of attack until of -2.5°. Moreover, the conventional configuration has a minimum drag near *alpha* = 0° which is a more desirable operational condition. This undesirable condition in the tandem wing could be changed with a redesign and an adjustment of the geometrical parameters, which can provide a minimum C_D closer to the most common operational condition and consequently boost the aerodynamic characteristics.

Finally, the tandem configuration presents higher values of the lift-to-drag ratio when only induced drag is accounted for but lower values when the parasite drag is included as is show in Fig. 6. This behavior is expected because the rear wing of the tandem aircraft is affected by the wake of the forward wing, reducing it is lifting characteristics, while the conventional one does not provide a significant lift force; hence the affectation is minimum to the aerodynamic efficiency. To avoid the aforementioned effect produced by the aerodynamic interference between two wings, optimization of some geometric parameters, such as gap, stagger, and dihedral angle, to improve aerodynamic efficiency could be performed.

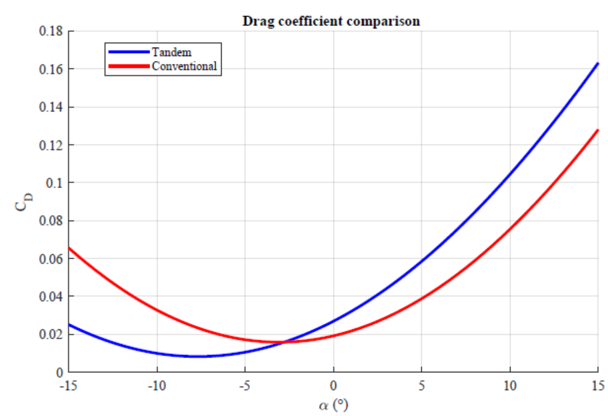


Fig. 5 Drag coefficient vs angle of attack for both configurations

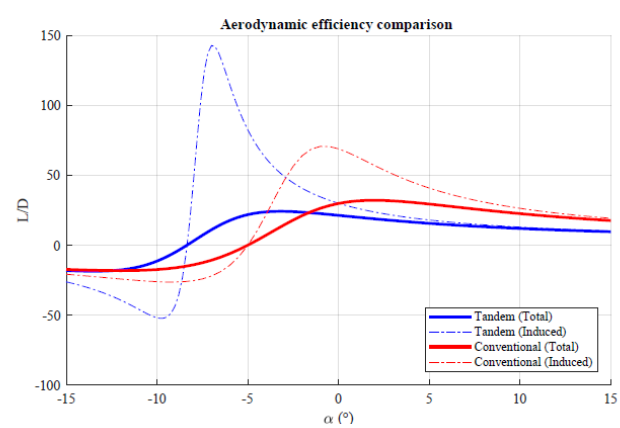


Fig. 6 Aerodynamic efficiency comparison.

IV. CONCLUSIONS

A 3D wing design is a process that can be achieved through several procedures that aim to calculate the aerodynamic characteristics required for the requirements of a specific mission. The named GHVM allows a model of a tridimensional wing layout with multiple lifting surfaces for the constraints of a specific and obtain its main aerodynamic characteristics, such as lift slope and induced drag, which can be completed by parasite drag empirical equations to obtain the curve of C_D vs *alpha*. Having these characteristics, the wing can be analyzed to check its behavior during the mission specified at the very first step of the preliminary design.

The advantage of the present results is the accuracy of the numerical method without having a high computational cost, providing results with higher accuracy than the Prandtl-Munk theory and vortex lattice predictions for the induced drag and with better results for the lift force calculations in comparison with LLT, which is obtained due to the consideration of the disturbance in the flow between the lifting surfaces of the system. In addition, the model's simplicity makes it possible to develop the calculations in a computing environment such as GNU Octave, consisting of a tool accessible to the general public. The method benefits low-cost projects where a wind tunnel test or CFD analysis cannot be afforded, but acceptable accuracy is needed.

The above analysis may not be true when the small disturbance, thin airfoil, large AR, and the attached fluid assumptions are violated. These conditions are required to simplify the airfoil and the wing so that the flow field is linear and the vortices can be modeled through the set of equations presented.

Finally, this method was applied to a UAV's design comparison between tandem and conventional to obtain each configuration's advantages and disadvantages, analyzing each design's behavior and concluding the benefits of each according to the specific application sought for the UAV.

Firstly, the conventional configuration shows a more stable behavior and a greater operating range due to its high α_{Stall}, unlike the tandem configuration; however, this latter allows the aircraft to descend more smoothly and to recover more easily from stall since the rear wing can generate lift when the front wing is stalling; secondly, both configurations lead to a trailing vortex of the front wing affecting the rear surface; however, the tandem has a higher negative impact since the secondary wing generates about 50% of the lift, unlike the conventional one; lastly, the difference in size is considerable and provides a greater range of applications to the tandem configuration where the size of the aircraft is crucial, such as environments with a high number of obstacles or very small passages.

Both tandem and conventional configuration have plenty of advantages suitable for different requirements. This study shows that the conventional wing configuration requires less analysis and provides good results; nevertheless, the tandem configuration has many aerodynamic characteristics that can be exploited and surpass the performance of the conventional wings if a thorough design is performed.

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